

1.4

Focus on Representing



Mathematics can be used to represent situations using diagrams, numbers, graphs, algebra, calculator applications, and computer software. In this section, you will develop ideas on how to represent both mathematical and real-life situations.

Investigate

How can you represent a situation numerically?

When Kevin was scuba diving, he entered a shipwreck and immediately dove down 6 m, came up 5 m, dove down 9 m, and then dove a further 2 m, to finish at 32 m below the surface of the water. What was Kevin's depth when he entered the shipwreck?

Understand the Problem

Choose a Strategy

Carry Out the Strategy

Reflect

1. Read the problem above. Read it again. Express the problem in your own words.
2. An effective strategy is to represent Kevin's movements as integers on a labelled diagram. Then, work backward from Kevin's finishing position.
3. Start at the finishing level and record Kevin's previous steps in reverse.
4. Verify that your results hold for Kevin's dive.

Example Represent a Situation Using a Diagram

Eight friends arrive at a party. Each person shakes hands with each other person once. How many handshakes occur?

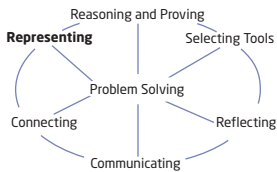
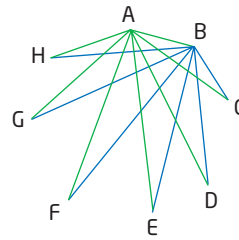
Solution

Illustrate the handshakes using a diagram. Place the eight letters from A to H in a circular pattern. Draw seven green line segments from person A to all the other friends, B to H.

Continue by drawing six blue line segments from B to the remaining friends, C to H. Continue the pattern. Find the total number of line segments.

$$7 + 6 + 5 + 4 + 3 + 2 + 1 = 28$$

There are 28 handshakes among eight friends.



Key Concepts

- A mathematical situation can be represented in many ways, including numerically, graphically, algebraically, and with a diagram.
- Drawing a diagram can be a useful strategy to help visualize the situation.
- Representing data numerically can help you see a pattern or a relationship between numbers.

Communicate Your Understanding

- C1** In the Investigate, you used the working backward strategy. Describe another strategy that you could use to determine Kevin's depth when he entered the shipwreck.
- C2** In a walk-a-thon, Ginny is 5 m behind Alice, who is 8 m ahead of Barbara. Sarah is 4 m ahead of Alice and 10 m ahead of both Ruth and Denise. What is the distance between the first and last of the girls? Describe how you would solve this problem.

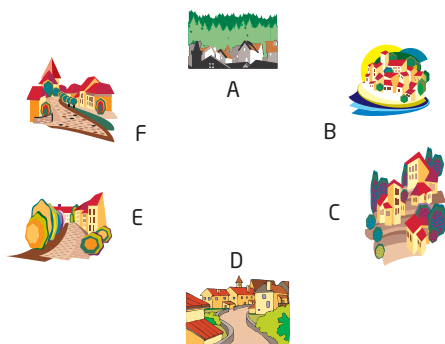
Practise

For help with question 1, see the Investigate.

- On a treasure hunt, players are instructed to go 2 km north, then 5 km east, then 4 km south, then 3 km west, and finally 2 km north. Where is the treasure relative to the starting point?

For help with questions 2 to 4, see the Example.

- The top floor of an apartment building has eight apartments. Each of the other floors below it has $1\frac{1}{2}$ times as many apartments as the floor above. Use a diagram and a numeric representation to help determine the maximum number of floors that this building can have.
- Seven friends have initials O, P, Q, R, S, T, and U. Each must have a telephone conversation with friends whose initials are within two letters of their own. Use a diagram and a numeric representation to determine how many telephone conversations will occur.
- A direct road needs to be built between each pair of the six towns shown. How many roads need to be built?

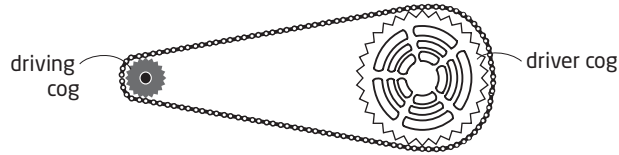


Connect and Apply

- Plot each set of points on a grid. Describe the pattern and plot the next three points.
 - A(2, 3), B(5, 4), C(8, 5)
 - P(1, 6), Q(-4, 4), R(-9, 2)
 - G(3, 3), H(0, 0), I(-3, -3)
- Use a diagram to show which fraction is greater. Describe the pattern in the two fractions being compared. Make a general statement about the pattern and which fraction is greater.

a) $\frac{1}{2}$ or $\frac{2}{3}$	b) $\frac{2}{3}$ or $\frac{3}{4}$
c) $\frac{3}{4}$ or $\frac{4}{5}$	d) $\frac{4}{5}$ or $\frac{5}{6}$

7. A bicycle gear ratio compares the number of teeth on the driver cog to the number of teeth on the driving cog. The driver cog on a bicycle has 30 teeth and the driving cog has 20 teeth.

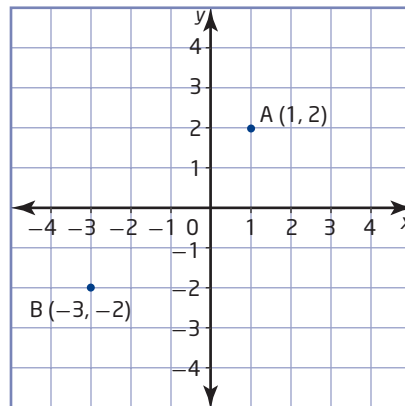


- If the driver cog rotates three turns, how many turns does the driving cog make?
- If the driver cog makes a half turn, how many turns does the driving cog make?
- How many turns of the driver cog are required for the driving cog to turn five times?
- On a different bicycle, the driver cog has 24 teeth and the driving cog has 40 teeth. If the driver cog makes a half turn, how many turns does the driving cog make?

vertex (pl vertices)

- a point at which two sides meet

8. The points $A(1, 2)$ and $B(-3, -2)$ are two **vertices** of a square. Find all possible locations of the other two vertices.



Extend

- The points $C(-7, -3)$ and $D(-1, -3)$ are two vertices of right $\triangle CDE$. Find all possible locations of the third vertex so that the area of the triangle is 15 square units.
- Use question 7 as a reference. How would you determine the resulting number of turns of the driving cog if another, middle, cog is added?

